1. Overview

Measurements of directional wave field are often required for a variety of applications including coastal engineering and near-shore dynamics, sediment transport and beach erosion, waste dispersal and pollution studies. In the past, specially designed equipment was often used to provide wave data, which often required cumbersome installation (capacitance wire gages), raised integration and data storage concerns (distributed pressure arrays), and amounted to significant costs (surface buoys). Earlier pressure-velocity (PUV) systems, while offering simplified deployments, lacked the accuracy required for robust operation. (due to electromagnetic and travel time velocity sensor drift, fouling, etc.)

Recently introduced ADCP-based beam systems are reported to be capable of gathering directional wave data. In this technique, velocity measurements are taken at locations that are separated in space by tens or hundreds of millions (Figure 1). The beam separation also limits the minimum wavelengths that can be resolved. ADCP-based systems or beam separation may also affect velocity data, especially if currents near the surface are spatially inhomogeneous. The Nyquist limit on wavelengths is that an array can resolve waves no shorter than twice the spacing between the adjacent measurement points. Most profilers have an off-vertical angle of 20-30 degrees, which means that a profiler deployed 50 m deep will create an array with measurement points a minimum of 73-115 m apart, making the minimum measurable wavelength 146-230 m. For reference, ten-second waves have wavelengths of 150 m.

This approach also relies on acoustic surface detection for wave height measurements, which may be tricky at best due to several factors. First, limited sampling bandwidth of the raw acoustic backscatter restricts the range resolution [Zedel, 1994, Polonichko, 1998]. This is especially pronounced in long-range systems. Second, the broadband (coded) pulses are not best suited for the task of surface detection [Zedel, 1994, Trevorrow, 1998]. Hence, use of separate narrowband pings may be required and which

![Figure 1: Deployment configuration and limitations of an ADCP-based beam system: Left: spatial beam divergence limits the minimum wave length that can be unambiguously resolved (the Nyquist limit). Right: beam spreading and side-lobe interference effects on data collection near a boundary. False surface is detected when the sidelobe return $A_{side}$ exceeds the main lobe signal $A_{main}$.](image)
will decrease the velocity sampling rate. Last, changes in the density of the water column (e.g. a thermocline) will affect the speed of sound, and therefore the associated level measurement.

In addition, the fact that the beams intersect the surface at an angle may produce significant signal reflections from the beam sidelobes (Figure 1, right) and therefore result in erroneous range to surface and contaminated velocity data near the surface (normally ~10% of the depth).

To compute wave directional spectra, the beam technique uses recently introduced experimental processing algorithms. Because of complexity, this processing may be difficult to debug. Although it can provide better angular resolution than the Parametric Spectral Method, described below, the beam technique relies upon the Maximum Likelihood Estimator (MLM). This estimator essentially acts as a spatial filter that rejects as much of the observed input as possible while passing a certain prescribed input un-attenuated [Davis and Regier, 1977]. This may explain why the MLM results tend to produce bi-directional distributions and the estimator often needs to be initialized with an independently measured wave distribution to produce robust results.

The advent of high accuracy acoustic Doppler sensors, such as the SonTek ADV Ocean, allowed velocity measurements in the oceanic environments with high temporal resolution and laboratory precision. Supplemented by high accuracy pressure sensors, these systems have been used extensively for directional wave measurements and are often employed as a reference to verify array and other custom systems.

Expanding on this capability, the SonTek SONWAVE-PRO wave data collection package allows users of the Acoustic Doppler Profilers, PC-ADPs, ADV Oceans, and the Triton wave/current/tide gauge to gather wave data (including the wave-frequency spectra, significant wave height, wave direction and wave peak period) with a single instrument using proven acoustic Doppler technology. The advantages presented here are:

- The data are gathered synchronously with the velocity and other measurements;
- No external hardware and/or wiring are required; deployment logistics are more simplified and costs are reduced;
- The wave-specific signal processing, based on the Parametric Spectral Method, is included;
- This processing requires very few additional assumptions and provides a stable solution, regardless of flow conditions;
- The analysis is fairly straightforward and can be easily checked;

Because PSM is essentially a first order technique, SonWave predicts very accurately the first order wave statistics: wave height, the peak period, the mean direction and spreading. This is sufficient for most of the engineering and monitoring applications. If higher order solutions are required, SonWave-Pro provides the user with a complete set of the raw data in either ASCII or Matlab format for further custom processing.

### 2. SonWave Measurement Technique

The SONWAVE technique relies on the high precision burst measurements of hydrostatic pressure (P) and horizontal water velocity (UV) to compute wave parameters using the Parametric Spectral Method (PSM). Velocity is measured with high accuracy, at a single location just above the instrument (Figure 2) at up to 4 Hz rates (Hydra and ADP). The PC-ADP is generally limited to a maximum data rate of 2 Hz because wave data is gathered over the entire profile rather than at a single point.

SonTek ADPs use state-of-the-art digital incoherent processing for measurement of velocity profiles. Although this technique works well for measuring mean currents, it can not produce velocity data with
sufficient accuracy and time resolution required for directional wave analyses. Instead, an ADP configured for directional waves, uses a Doppler technique known as “Pulse Coherent Processing” to measure bursts of velocity at short range. This is the same highly accurate technique that was originally implemented in the SonTek Acoustic Doppler Velocimeter (ADV) [Cabrera and Lohrmann, 1993, Kraus, 1994], the Hydra [Gelfenbaum et al, 2000, Sherwood et al, 2000] and PC-ADP systems [Polonichko et al 1999].

By confining the range to a single, nearby cell, the ADP can easily obtain required velocity measurements using the highly accurate pulse-coherent technology [Zedel et al. 1996] employed in the Hydras and the PC-ADPs. A typical sampling strategy for an ADP, equipped for directional wave measurements, might be a five-minute measurement of the velocity profile every 30 minutes with bursts of the pressure and short-range velocity measurements interspersed every 4 profiles (2 hours).

3. Directional Wave Signal Processing

Below we describe basic principles that are used by SONWAVE-PRO for computing wave parameters from a pressure-velocity time-series.

3.1. Surface Gravity Waves

Surface gravity waves are characterized by the directional wave spectrum, which is commonly expressed as a composition of the frequency spectrum $S(f)$ and directional spreading $D(f, \theta)$:

$$S(f, \theta) = S(f) \times D(f, \theta).$$

where $D(f, \theta)$ satisfies the following norm:
\[
D(f, \theta) d\theta = 1.
\]  

(2)

The frequency spectrum is normally derived for the time-series of surface elevation (gathered with either the pressure sensor or range to surface), while the directional spreading is computed using the pressure-velocity data.

A wave of amplitude \(A\) and frequency \(\omega\) propagating in the \(x\) direction induces hydrostatic pressure perturbations, which decay with depth as

\[
p(\omega, z) = A \rho_w g \frac{\cosh(k(z + H))}{\cosh kH} \cos(kx - \omega t) 
\]

and a corresponding velocity field

\[
[u(\omega, z), v(\omega, z)] = A \omega \frac{\cosh(k(z + H))}{\sinh kH} [\cos(kx - \omega t), \sin(kx - \omega t)] .
\]

3.1.1. Accounting for Mean Current

In the presence of strong ambient currents \(U\), the general wave dispersion relation (Eq.4) needs to be modified to account for the Doppler shift in the frequency of the waves propagating through a moving medium:

\[
\omega = \sqrt{g k \tanh kH} + kU \cos \alpha ,
\]

where \(U\) is the current magnitude and \(\alpha\) is the angle between the wave and the current direction. This correction becomes significant when the Doppler term is comparable with the general dispersion term in (Eq. 4). Therefore the criterion for taking the mean current into account is

\[
U \cos \alpha \geq 0.14 \sqrt{\frac{g}{k}} \tanh kH .
\]
3.2. Parametric Spectral Method

To estimate wave directional distribution $D(f,\theta)$, we at SonTek employ the Parametric Spectral Method (PSM) as initially described by Longuet-Higgins et al. [1963] for a wave buoy and generalized to a generic 3-component sensor by many others (see Grosskopf, [1992] for more detail).

In the PSM description, the wave directional function $D(f,\theta)$ is expanded into its Fourier equivalent with the amplitudes of the Fourier coefficients determined from auto-spectra and co-spectra of the pressure and velocity time series. In general, the first five Fourier coefficients are taken to adequately represent the wave field, and are:

$$a_0(\sigma) = \frac{1}{2\pi R_u^2 \rho g \sigma} S_{pp}(\sigma) = \frac{1}{2\pi R_u^2 \sigma^2} (S_{uu}(\sigma) + S_{vv}(\sigma))$$

$$a_1(\sigma) = \frac{1}{\pi R_u R_p \rho g \sigma} S_{pu}(\sigma)$$

$$a_2(\sigma) = \frac{1}{\pi R_u^2 \sigma^2} (S_{uu}(\sigma) - S_{vv}(\sigma))$$

$$b_1(\sigma) = \frac{1}{\pi R_u R_p \rho g \sigma} S_{pv}(\sigma)$$

$$b_1(\sigma) = \frac{1}{\pi R_u^2 \sigma^2} S_{uv}(\sigma)$$

Where $S_{ii}$ is the auto-spectrum of variable $i$, and $S_{ij}$ is the co-spectrum of variable $i$ with variable $j$. $R_u$ and $R_p$ are normalization factors to correct for the known decay of pressure and wave orbital velocities with depth, and are also used in the standard linearized form:

$$R_u = \frac{\sinh k(z + H)}{\cosh kH} \quad R_p = \frac{\cosh k(z + H)}{\cosh kH}$$

These corrections for depth define the depths to which the pressure and velocity time series can be expected to penetrate for a given wave frequency. For example, the pressure signal from a 10-s wave is reduced by more than 95% when it reaches a sensor at 75-m depth. The actual amplitudes that can be measured at depth depend on their surface amplitudes in addition to their frequency, so it is difficult to determine a precise relationship between a wave’s period and the depth to which it can be measured. As a general rule of thumb, an instrument deployed at depths of up to 20 m can reasonably be expected to detect waves of five seconds or longer. In the rare event that shorter wave periods are of interest, the instrument must be deployed at a shallower depth.

3.3. Other processing Techniques

The Parametric Spectral method described above is a proven technique that has been used extensively for the past several decades. PSM requires very few assumptions and provides a stable solution, regardless of flow and wave conditions. The analysis is straightforward and can be easily checked. Because this is
essentially a first order technique, the PSM predicts very accurately the first order wave statistic: the significant wave height, peak period, mean direction and spreading. This is sufficient for most engineering and monitoring applications.

If a higher order solution is required, the PSM processing can be augmented by a choice of an autoregressive (Bayesian) technique such as the Maximum Entropy Principle (MEP) or Extended Maximum Entropy Principle (EMEP) [Hashimoto, 1988]. SonWave-PRO provides the user with a complete set of the raw data in either ASCII or Matlab format for further custom processing.

4. Experimental Verification

To verify performance of the SonWave directional wave capability, two separate deployments were conducted off Scripps Pier San Diego in November 1999 and November 2000. The first deployment was with a PC-ADP and an ADV Ocean (Hydra) during which a few bursts of data were collected. The second test lasted for about a week where a standard ADP equipped with SonWave-PRO was deployed along with a Hydra system. The main purpose of the first test was to verify the pulse coherent profiling accuracy, while the second test was intended for wave data collection in the near-shore.

During the first deployment, both systems were mounted on a frame at about 1.20m above the sea bed. Moderate wave conditions were encountered with the wave orbital velocities in excess of 120 cm/s. Detailed description of the PC-ADP data is given by Polonichko et al. [2000] and Polonichko and Cabrera, [2000].

In the second test, a 1.5 MHz ADP and the ADV Ocean were deployed on the bottom as shown in Figure 3. The ADVO was equipped with a 20-m resonance crystal pressure transducer (RPT), which has 0.01% accuracy and better than $10^{-6}$ stability. The system was programmed to collect a 17-min long time series at 2-Hz twice each hour. The ADP was set to collect 5-minute mean current measurements followed by a 17-minute burst pressure-velocity series gathered at 2 Hz.

The ADP wave sampling cell was set at 20cm above the instrument. The coherent pulse had a lag of 0.5m, which in combination with a 15-cm long cell delivered a mm/s velocity precision at 2 Hz (refer to SonTek PC-ADP Principles of Operations for a detailed description of the pulse-coherent Doppler processing). The ADP was outfitted with basic strain gage pressure sensor (1% accuracy), which had a 100-m full scale mainly due to sensor availability.

The CPU clocks of the two instruments were synchronized to within 1-s and subsequently the time series were re-aligned by 2 samples (1-s) using the pressure gradient, which occurred when both instruments entered the water. Both systems were oriented such that their X-axes were aligned to within 3° in order to simplify subsequent comparison of the data regardless of the compass orientation.

The wave conditions were weak to moderate with the significant wave height ranging from 60 to 20 cm.

![Figure 3: Test deployment configuration. The ADV Ocean and the ADP were set such that their sampling volumes were located at the same level.](image)
140 cm, with orbital wave velocities reaching 100 cm/s, and the water depth varying between 4 and 7 m due to tides.

4.1. Velocity Comparison

To measure velocity, the SonTek Hydra system employs the Acoustic Doppler Velocimeter (ADVOCEAN), which features laboratory level precision [Kraus et al, 1994, Nikora and Goring, 1998] and hence serves as an excellent tool for wave velocity measurements [Sherwood et al, 2000, Gelfenbaum et al, 2000]. An ADV samples three velocity components in a single location (sampling volume), which is located approximately 18-cm above the probe tip (Figure 3). Internally, an ADV samples velocity at a very high rate (between 150 and 200 Hz); since all three velocity components are measured in the same location, no assumptions of flow homogeneity are required.

Comparison of the on-shore velocity time series (Figure 4, left) shows excellent agreement between the ADP and Hydra data. This is quite remarkable, given the facts that the two sampling volumes are of different size and that ADP samples velocity over locations that are 0.5 m apart. Comparing the spectra for the same data segment (Figure 4, right), we see that both instruments resolve waves, up to 4-s (0.25 Hz), with the ADP spectrum hitting the noise floor at approximately 0.32 Hz. The ADV spectrum is valid to about 0.48 Hz (2-s waves) which is expected due to faster sampling and a smaller sampling volume. To conclude, these tests verify that the ADP operating in pulse coherent mode can sample velocity accurately enough to resolve 4-s waves in 7.5 m of water. Note, the velocity data presented above were collected at the beginning of the deployment when the significant wave period was the lowest. Performing the same comparison for a more energetic period (burst 110 on November 22, Hs~140 cm), we find that the ADP frequency response does not change significantly, although the spectral energy level is increased. We can therefore conclude, that the limiting factor in the ADP spectral resolution is beam spreading rather than sampling accuracy.

The high resolution velocity data collected with the PC-ADP during the November 1999 test allow verification of another important aspect of the wave phenomena: wave energy depth decay. That is demonstrated to great effect in Figure 5, which displays the wave velocity spectra for each horizontal layer measured by the PC-ADP and directly shows the expected decay rate, consistent with the linear theory.

![Figure 4: ADVO velocity comparison. Left, velocity time series show excellent agreement between the ADP and ADVO. The corresponding spectra (right) show that the ADP resolves waves down to 0.33 Hz (3-s) scales, while the ADVO cut off is at ~0.45 Hz. Mean depth was 7.5 m and Hs=62 cm.](image-url)
4.2. Wave Height Comparison

To verify the pressure sensor response to wave forcing, we compare the data collected by the ADVO, the ADP, and independent measurements collected as part of the ongoing wave monitoring at Scripps Pier (the data are commonly available via the Scripps WWW). The Scripps station uses a digiquart pressure sensor placed at approximately 7-m depth, which has accuracy and stability specifications similar to the RPT.

Comparing the ADVO and the ADP pressure time series (Figure 6, left), we find that while the two are in general agreement, the ADP record displays the quantization signature due to the fact that a 100-m sensor was deployed in 7-m of water. However, comparison of the corresponding spectra (Figure 6, right) shows that even the 100-m sensor is capable of detecting the wave signal down to 3-s (0.3 Hz) scales. The RPT sensor offers better frequency response with a cut-off at about 0.45 Hz, which is close to the velocity...
We conclude that both sensors do a good job at resolving the waves of interest, with the RPT sensor providing a better match to the ADVO velocity resolution capability.

To verify the wave height measurements, we compute the significant wave height $H_{mo}$. A generally accepted estimate of the significant wave height can be easily obtained from the spectral amplitudes using:

$$H_{mo} = 4\sqrt{\sigma}^2,$$

where $\sigma$ is the total energy defined as

$$Total\ Energy\ (\sigma_i^2) = \sum_{i=1}^{10} A_i^2$$

We further compare the significant wave height computed by SONWAVE-PRO with the independent measurements collected by the Scripps monitoring program. As seen from Figure 7 the SonWave data are in excellent agreement with the Scripps estimates in a variety of wave conditions ranging from fairly calm at the beginning of the test # ($H_s=60$ cm) to moderate on November 23 ($H_s\approx 140$ cm/s).

### 4.3. Wave Directional Distribution

Unfortunately, the wave-monitoring program at Scripps does not include directional information. Because the shoreline orientation at this location, a generally eastward wave direction is observed. Example stet the wave directional energy distribution as a function frequency (Figure 8) shows waves propagating east with most of the wave energy is concentrated at approximately 0.15 Hz.

The data presented in Figure 9 give an example of other wave phenomena that can be analyzed with SONWAVE. The velocity spectra calculated from the ADP wave data are plotted with the mean tidal series to show clearly that the tides are modulating the wave field. As the tide flows, the effective instrument
depth (the height above the measurement point) increases, hence resulting in a greater wave energy attenuation at higher frequencies. At low tides the attenuation is decreased, and shorter waves (higher frequency components) are detected by the ADP.

The bottom plot in Figure 9 displays the mean wave directional distribution as a function of time, which also shows dependence on the tide. Comparison of these data with the wave activity (significant wave height, top plot in Figure 9) reveals the broadening of the wave directional distribution when the wave activity is increased on November 23 and 26. This is a commonly observed feature of the near-shore wave dynamics.

5. Conclusions

The SONWAVE-PRO directional wave data collection package offers a turnkey solution to the wave monitoring requirements in near-shore applications. Provides key wave parameters such as the wave height period and directional energy distribution without requiring additional computations. This information is collected along with our well-respected current measurements using the same simple, turnkey system. The performance of the system has been experimentally verified and field tests show that an ADP equipped for wave data collection can resolve waves down to 3-s, and the measurements agree remarkably well with that of an ADV. The tests also confirmed that a modern pressure sensor is capable of capturing the wave scales of interest and that wave height computations agree well with independent measurements. For additional information, please contact SonTek.

Figure 8: Distribution of the wave energy as a function of direction and frequency showing predominately eastward wave propagation consistent with geographic observations.
Figure 9: Time evolution of the ADP velocity spectra show (middle plot) modulation of wave energy that is consistent with the tidal record (white line). The bottom plot displays mean wave directional distribution as a function of time. It shows dependence on tide as well as directional broadening coinciding with the high wave activity.

6. References


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